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DETERMINATION OF PRESSURE IN A TWO-PHASE WATER SYSTEM

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Abstract

A method of determining pressure changes in a two-phase water system is presented. Two applications are given which pertain to the design of nuclear reactor systems. The method utilizes information obtained from simple calculations to enter a chart from which the pressure can be readily determined. In addition, an analytical approach is taken and an equation derived which describes pressure changes also, but less accurately than the graphical method.

Nomenclature

a	- constant, defined as used
b	- constant, defined as used
c	- constant, defined as used
c_v	- specific heat of air at constant volume, Btu/lb-°F
d	- constant, defined as used
h	- enthalpy, Btu/lb
lb	- pound mass
m	- constant, defined as used
n	- constant, defined as used
P	- pressure, #/in ² absolute
Q	- energy added by heat transfer, Btu
t	- temperature, °F
u	- internal energy, Btu/lb
v	- specific volume, ft ³ /lb
V	- internal volume of a vessel, ft ³
w	- mass, lb
#	- pound force
Subscripts	
amb	- initial ambient value
av	- average
f	- refers to liquid phase
fg	- property of vaporization
g	- refers to vapor phase
in	- mass or energy entering
out	- mass or energy leaving
P	- at constant pressure
v	- at constant internal energy
s	- at constant specific volume
1	- initial state
2	- final state

Graphical Method

It is well known that if two intensive thermodynamic properties of a pure substance in equilibrium are known, then all the other like properties are fixed and can be found as functions of the known properties. The pure substance examined herein is water in two phases (liquid and vapor) which are in equilibrium. The phases are contained in a rigid, non-adiabatic vessel which is connected to sources and sinks of the same substance not necessarily saturated. The initial amounts of liquid and vapor are known as well as the initial pressure. The assumption is made that electricity, magnetism, capillarity, changes in elevation, and kinetic energy are negligible. Consider now that some fluid leaves the vessel (composed of either or both phases) and is replaced in part or in whole by a mass of liquid or vapor in an indefinite amount of time. Referring to Figure 1, and treating the vessel as a control volume, the following balances may be written:

Energy Balance

$$w_{f2}u_{f2} + w_{g2}u_{g2} = w_{f1}u_{f1} + w_{g1}u_{g1} + w_{in}h_{in} - w_{out}h_{out} + Q_{in} \quad (1)$$

Mass Balance

$$w_{f2} + w_{g2} = w_{f1} + w_{g1} + w_{in} - w_{out} \quad (2)$$

Volume "Balance"

$$w_{f2}v_{f2} + w_{g2}v_{g2} = w_{f1}v_{f1} + w_{g1}v_{g1} = V \quad (3)$$

One method of solving for the final pressure is by the cut-and-try process. A final pressure is assumed and the values of internal energy and specific volume are gotten from a table of properties such as reference (1). Equations (1) and (3) can then be solved for w_{f2} and w_{g2} . Equation (2) is then used as a check on the correctness of the assumed pressure.

This process can be reduced to a straight-forward solution by utilizing the definition of steam quality, "x".

$$x \equiv w_g / (w_f + w_g) = w_g / w \quad (4)$$

From this definition, the following is derived:

$$w_g = xw \quad (4a)$$

$$w_f = (1 - x)w \quad (4b)$$

Furthermore,

$$u = (1 - x)u_f + xu_g \quad (5)$$

and,

$$v = (1 - x)v_f + xv_g \quad (6)$$

Utilizing equations (4a), (4b), (5) and (6), equation (1) can be rewritten as:

$$w_2 u_2 = w_1 u_1 + w_{in} h_{in} - w_{out} h_{out} + q_{in} \quad (7)$$

Similarly, equation (2) can be written:

$$v_2 = v_1 + w_{in} - w_{out} \quad (8)$$

Equations (7) and (8) can be solved and combined to find u_2 :

$$u_2 = w_2 v_2 / w_2 \quad (9)$$

Also:

$$v_2 = v / w_2 \quad (10)$$

We now have two intensive properties and know that:

$$P_2 = P(v_2, u_2) \quad (11)$$

By entering the u - v chart (Figure 2) with the values of u_2 and v_2 , the pressure can be determined.

Example I - Pressurizer Outsurge.

To illustrate the use of the foregoing method, let us assume that we wish to know how far the pressure in a pressurized water reactor system will fall during an increasing power demand transient. The mass of liquid which leaves the pressurizer will be equal to the mass required to make up the contraction in the main coolant system. During rapid transients, the effect of pressurizer heaters is negligible and equation (7) becomes:

$$w_2 u_2 = w_1 u_1 - w_{out} h_{f,av} \quad (12)$$

where $h_{f,av}$ is the average enthalpy of the pressurizer liquid during outsurge. This average value can usually be approximated quite closely, but if very accurate results are desired, two or more iterations will be necessary. Equation (8) becomes:

$$w_2 = w_1 - w_{out} \quad (13)$$

Dividing (12) by (13) gives v_2 . Dividing the volume of the pressurizer by the result of (13) will yield v_2 . The final pressure is found by entering Figure 2 with these quantities.

Example II - Vapor Container

The pressurized water reactor system of the present day is always designed to be installed in an enclosure which will contain the vapor resulting from a loss of coolant accident. The usual design considers that all the fluid in the primary system is lost as well as the fluid in one or more of the associated steam generators. In general, there are five fluids to be considered, namely:

1. Primary coolant in the main coolant system and auxiliary primary systems.
2. Primary liquid in the pressurizer.

-
- 3. Primary vapor in the pressurizer.
 - 4. Secondary liquid in the steam generator(s).
 - 5. Secondary vapor in the steam generator(s).

It is usually assumed that:

- 1. No condensation of vapor takes place on the walls of the containment vessel.
- 2. No energy is added to the vapor from hot metal in the plant.
- 3. The resulting liquid and vapor phases are in equilibrium with each other and with the air in the vessel.
- 4. All the fluid considered is released instantaneously.
- 5. No vapor or air escapes from the vessel.

The energy balance for the resulting liquid-vapor-air system may be written as follows:

$$w_2 u_2 = \sum w_i (u_i \text{ (of each fluid)} - w_{\text{air}} c_v (t_2 - t_{\text{amb}})) \quad (14)$$

where t_2 is the saturation temperature corresponding to the final partial pressure due to steam. The mass balance is:

$$w_2 = \sum w_i \quad (15)$$

It is necessary to start with a vapor container of given design and compute its void volume, V_f . The final internal energy and specific volume are computed as before and the final pressure of the vapor obtained from Figure 2. To this must be added the partial pressure due to the heated air:

$$P_{2,\text{air}} = 14.7(t_2 + 460)/(t_{\text{amb}} + 460)$$

The total absolute pressure is then the sum of the partial pressures due to vapor and air.

It would seem from equation (14) that the final pressure must be known beforehand in order to have a value of t_2 and that therefore a cut-and-try process must be used. This is true only if precise answers are desired. Practically, vapor containers are designed for low pressures of the order of 50 to 100 psi so that a good approximation may be made to t_2 . Furthermore, the second term on the right-hand side of (14) is very small compared to the energy in the plant fluids. Therefore a rather large error in estimating the final temperature would have to be made in order to significantly affect the result.

Analytical Approach

Because pressure is a point function, its differential is exact. We can therefore write from (11) the following:

$$dP = (\partial P / \partial u)_v du + (\partial P / \partial v)_u dv \quad (16)$$

From the calculus we know that:

$$(\partial P/\partial u)_v (\partial u/\partial v)_P (\partial v/\partial P)_u = -1 \quad (17)$$

or,

$$(\partial P/\partial u)_v = -(\partial v/\partial u)_P (\partial P/\partial v)_u \quad (18)$$

Also, for saturated states:

$$(\partial v/\partial u)_P = v_{fg}/u_{fg} \quad (19)$$

Utilizing equations (18) and (19), equation (16) may be written:

$$dP = (dv - [v_{fg}/u_{fg}] du) (\partial P/\partial v)_u \quad (20)$$

Integrating (20) from state 1 to state 2 and assuming that $(\partial P/\partial v)_u$ is a constant:

$$\Delta P = (\Delta v - [v_{fg}/u_{fg}] \Delta u) (\partial P/\partial v)_u \quad (21)$$

Practically speaking, equation (21) is of limited usefulness because of the assumption that $(\partial P/\partial v)_u$ is constant. An examination of Figure 3 will reveal that this property varies markedly even for relatively small changes in pressure. It is possible however, to linearize over useful ranges by writing:

$$(\partial P/\partial v)_u = a + bP \quad (22)$$

Substituting (22) into (20) and integrating from states 1 to 2:

$$P_2 = (a/b + P_1) e^{(\Delta v - [v_{fg}/u_{fg}] \Delta u)b} - a/b \quad (23)$$

The properties v_{fg} and u_{fg} are to be evaluated at the initial pressure. $(\partial P/\partial v)_u$ is to be evaluated at the final internal energy. For an explanation of this, the reader is referred to Figure 4 and the following paragraph.

Examination of equation (21) shows that the term $-(v_{fg}/u_{fg})(\Delta u)$ computes $v_2'' - v_1'$ and, since pressure is constant, defines state point $1''$. Multiplying $(v_1'' - v_1')$ by $(\partial P/\partial v)_u$ results in $(P_1 - P'')$, and defines state point $1'$ since internal energy is held constant at v_2 . The term $\Delta v(\partial P/\partial v)_u$ gives $P' - P_2$. The sum of these two differences is then $P_1 - P_2$.

Observing that $(\partial P/\partial v)_u$ vs P is nearly linear on the log-log plot of Figure 3 suggests that a more general equation could be written by letting this property assume the form:

$$(\partial P/\partial v)_u = aP^b \quad (24)$$

It is evident from Figure 3 that "a" and "b" are themselves functions of the internal energy. These can be represented, to a good approximation, by:

$$a = mu + c \quad (25)$$

and,

$$b = nu + d \quad (26)$$

Substituting (24), (25) and (26) into (20), evaluating m , n , c , and d from Figure 3, and integrating, the following equation is obtained:

$$P_2 = \left[P_1 \frac{-1.4 + 4.5 \times 10^{-4} u_2}{(-1.4 + 4.5 \times 10^{-4} u_2)/(2.0 - 5.0 \times 10^{-3})} \right]^{1/(-1.4 + 4.5 \times 10^{-4} u_2)} \quad (27)$$

Equation (27) is rather cumbersome for manual calculation, but could be useful in computer solutions where charts obviously cannot be used. The constants used in (27) were computed on the basis of a best fit for Figure 3 over its entire range. Better approximations could be made by limiting the range to one of interest for a particular problem. Figures 2 and 3 were drawn on the basis of properties given in reference 1.

Reference

1. "Thermodynamic Properties of Steam", J.H. Keenan and F.G. Keyes, John Wiley & Sons, Inc.

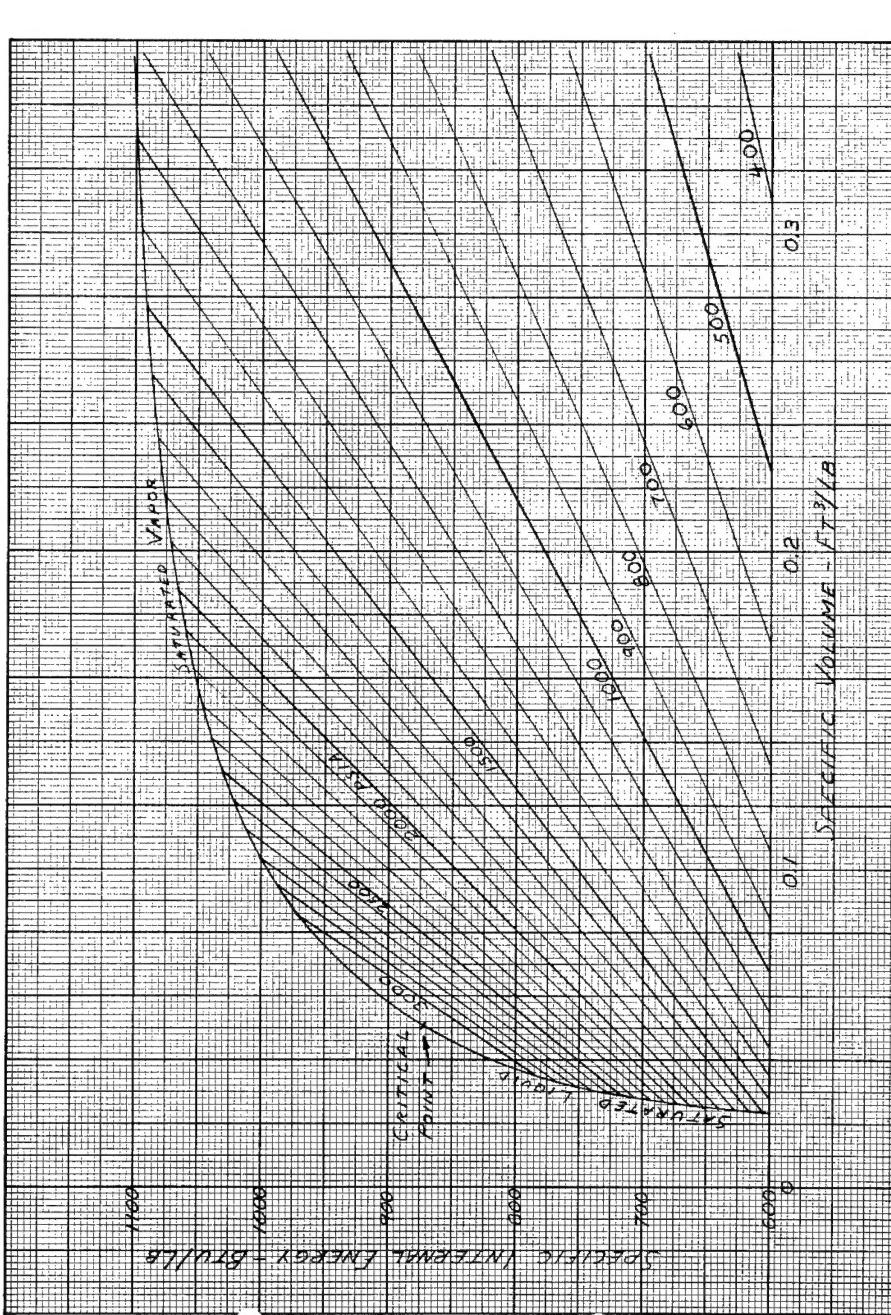
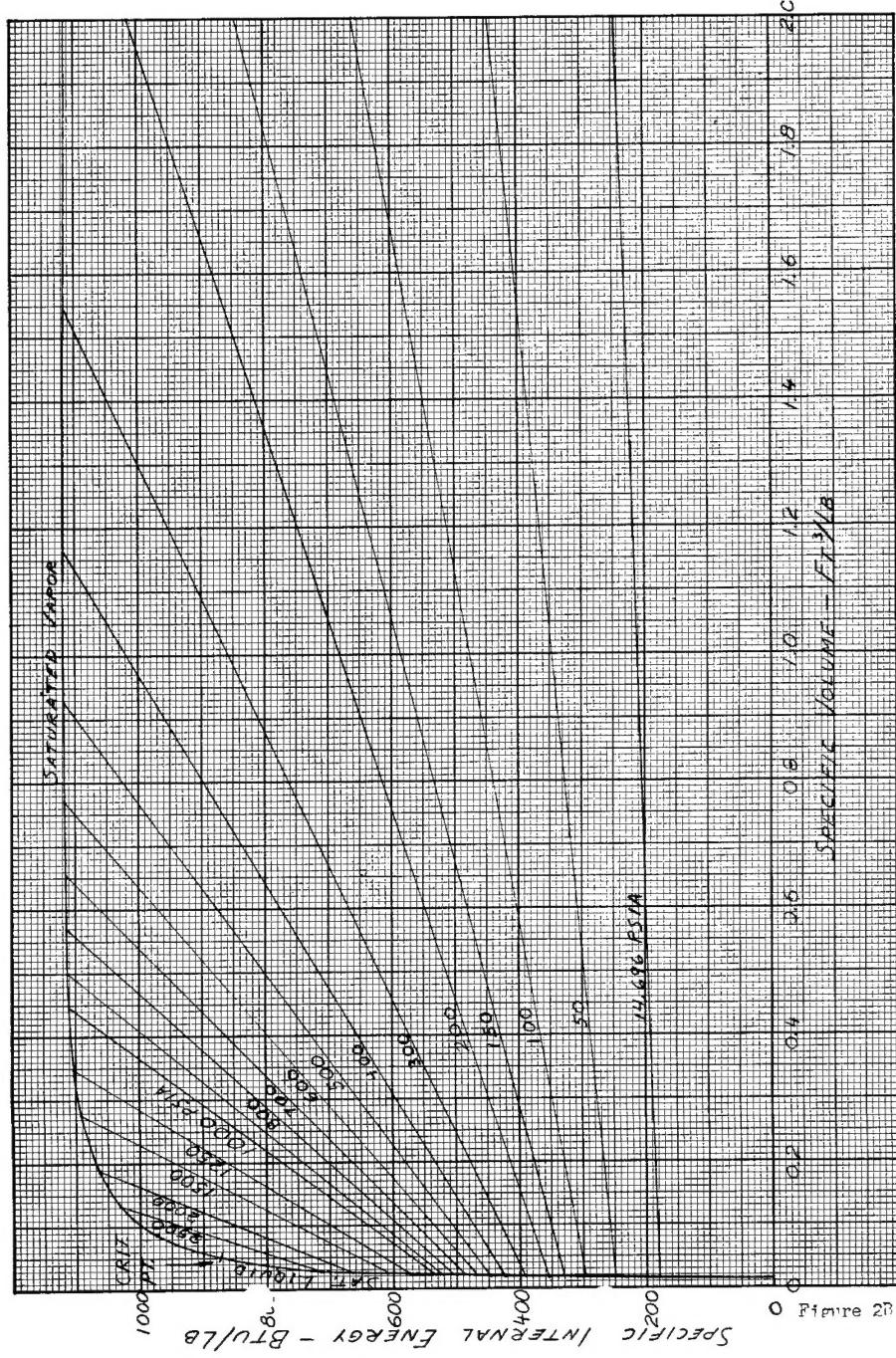


Figure 2A



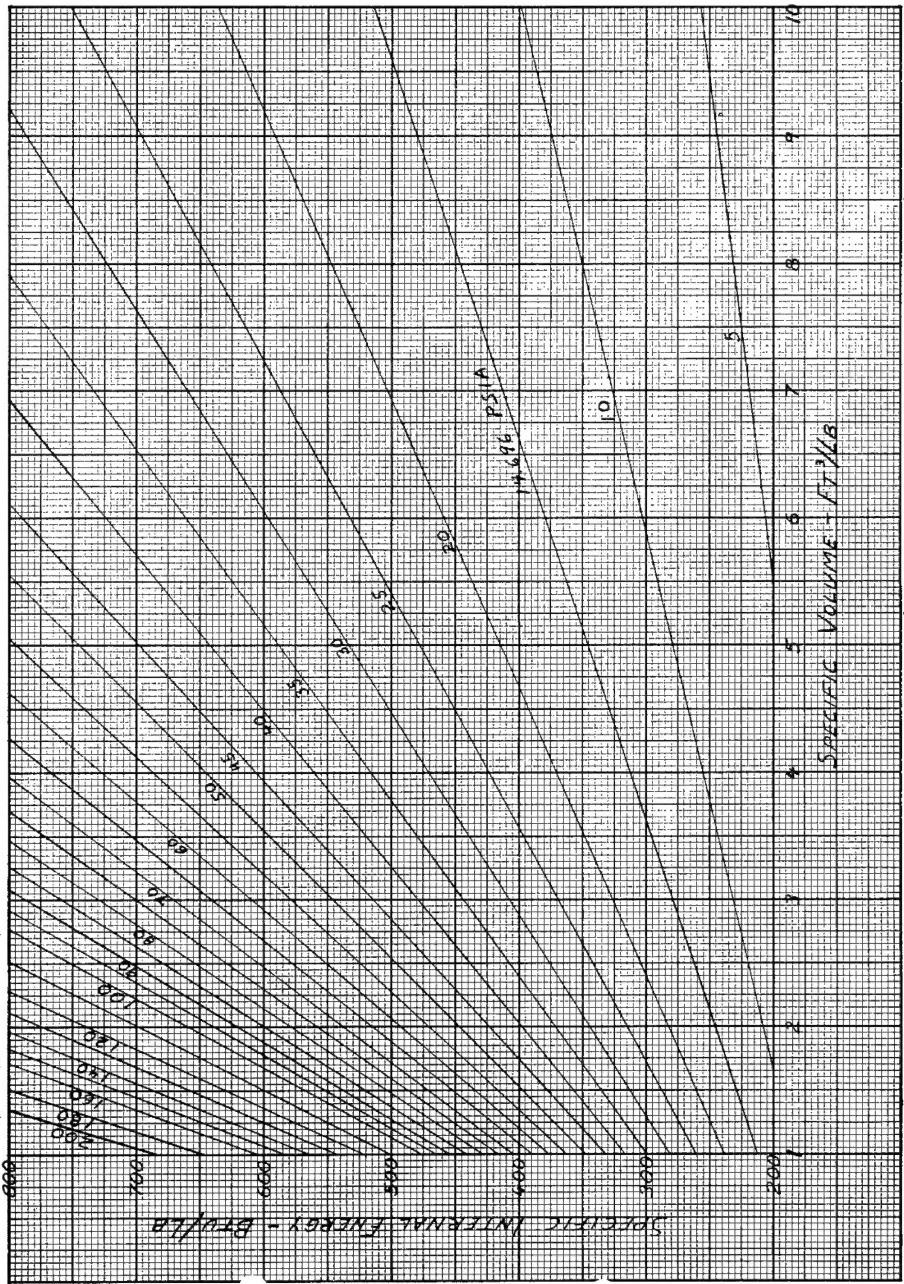


Figure 2C

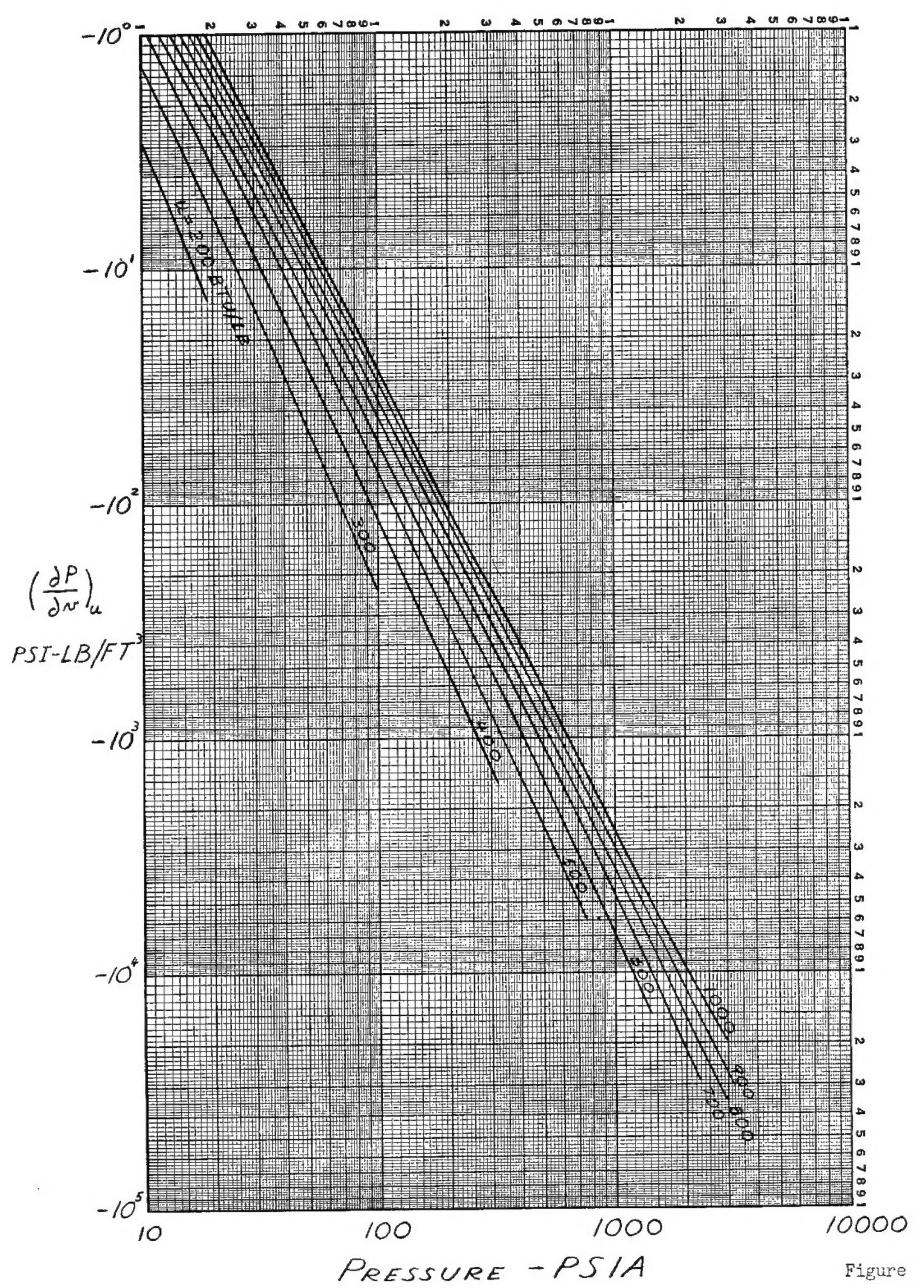


Figure 3

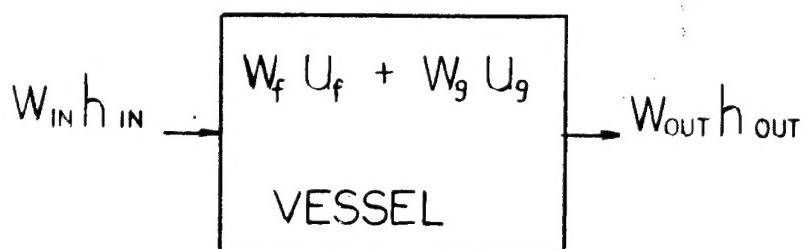


FIGURE I

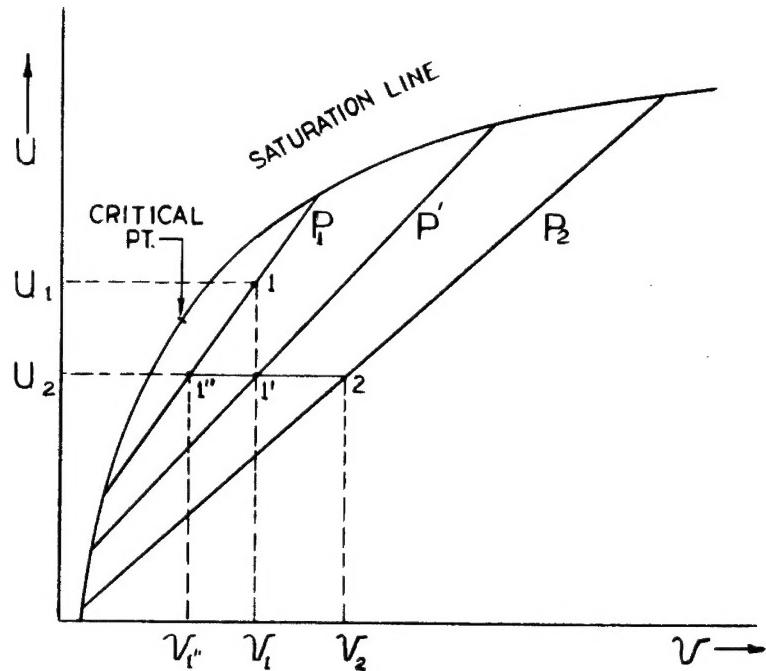


FIGURE 4